# EFFICIENT STOCHASTIC REPRESENTATION OF EARTHQUAKE GROUND MOTION

by

Erik H. Vanmarcke
Massachusetts Institute of Technology

## ABSTRACT

The paper proposes a stochastic representation of earthquake ground motion as a supplement and an alternative to conventional response spectra for purposes of seismic analysis and design. In the model the strong motion duration captures the essential transient character of earthquake ground motion, while the spectral density function $G(\omega)$ represents its "equivalent stationary" frequency content. This representation permits effective use of methods of random vibration to predict seismic response of many types of structures, and yields analytical predictions of response spectra. It provides a starting point for modeling the space-time variation of earthquake ground motion and offers an attractive format for bringing recent geophysical information about source parameters and ground motion frequency content to bear on earthquake engineering practice. It also facilitates accounting for the effects of local geology on ground motion. Finally, the proposed representation is well suited for use in seismic risk analysis and quantification of reliability under seismic loads.

## INTRODUCTION: THE CASE AGAINST RESPONSE SPECTRA

The most common representation of earthquake ground motion for seismic analysis and design is the response spectrum. Response spectra are plots of the maximum seismic response of a linear oscillator in function of the natural frequency for different damping ratios. They permit the designer to assess the severity of ground shaking directly in terms of the response of different alternative (simple linear) systems. The response spectra reflect the frequency content and the duration of the ground motion, as well as the way the motion is filtered by a single-degree linear oscillator.

The information content and the usefulness of response spectra are much reduced when the system of interest does not act as a simple linear oscillator. For linear multi-degree systems, one must resort to approximate rules for modal combination of response spectra ordinates at the different natural frequencies. Since the time interval during which strong ground shaking lasts is not explicitly accounted for, any phenomenon that is sensitive to motion duration tends to be poorly predicted by procedures based on response spectra, e.g., when inelastic action, low-cycle fatigue or liquefaction dominate behavior. It is also cumbersome to modify response spectra where it is necessary to account for local soil effects.

Further complications arise when seismic analysis must proceed on the basis of "design" response spectra which are, in a crude sense, envelopes of the response spectra corresponding to different types of possible ground motions (with different magnitudes and distances, and hence durations and spectral parameters). For example, an unknown degree of conservatism enters into the analysis of multi-degree linear systems when the modal ordinates of "design" response spectra - unlikely to occur simultaneously - are combined.

Another important weakness stems from the fact that the maximum ground acceleration (or the "zero-period" acceleration) is widely used as a scaling factor for acceleration time histories and response spectra. This has led to the development of "standard" response spectra (such as the Newmark-Blume-Kapur spectra) obtained from statistical analysis of a suite of recorded accelerograms scaled to a common maximum acceleration. It is well known, however, that the maximum acceleration is a rather unreliable indicator of ground motion severity for many kinds of systems, as it is very sensitive to (poorly known details of) the high frequency content of the ground motion. The high frequency components of the ground motions tend to be weakly correlated in space, and their effect on actual structures (with spatially extended foundations) may be much smaller than would be inferred from recorded accelerograms and their response spectra.

To date, it has been common in earthquake engineering to focus attention on the time history of ground acceleration (and its response spectrum) at a given location in space. This is a logical consequence of the fact that much of our knowledge about earthquake ground motion comes from recorded accelerograms, and that engineering attention has traditionally focused on "point" facilities for which it seemed reasonable to ignore "local" spatial variation of ground motion in seismic analysis and design. However, for spatially extended structures such as pipelines and embankments, or structures on widely-separated multiple supports or on large foundation slabs, the <u>spatial</u> variation of ground motion may be just as important as the <u>temporal</u> variation. Empirical strong-motion data from closely spaced arrays of seismographs is now gradually becoming available, and engineers are increasingly directing their attention to the effects of earthquakes on spatially distributed systems. It is evident that the response spectra format of motion representation is poorly suited to meet the challenge of accounting (for engineering purposes) for critical new information about spatial variability of earthquake ground motion.

The main purpose of this paper is to substantiate a proposal for the use of a direct stochastic representation of earthquake ground motion (in terms of the ground motion spectral density function, $G(\omega)$, and the duration of strong shaking, $s$) in earthquake engineering practice. While essentially equivalent to response spectra in reference to single-degree linear systems, the proposed stochastic representation leads to improved predictions of the response of linear multi-degree systems and the behavior of a variety of nonlinear systems, including those sensitive to low cycle fatigue and liquefaction. It also provides a tractable format for (a) dealing with the effect of spatial variation of ground motion, (b) accounting for the influence of local geology, and (c) relating ground motion frequency content (and duration) to basic earthquake source parameters and source-to-site distance.

## EMPIRICAL BASIS FOR THE PROPOSED MODEL

The Fourier amplitude spectrum of an accelerogram $a(t)$ is the absolute value of its Fourier transform:

$$A(\omega) = \left|\int_{-\infty}^{\infty} a(t)\, e^{-i\omega t}\, dt\right| = \left|\int_{0}^{t_0} a(t)\, e^{-\omega t}\, dt\right| \qquad (1)$$

in which $\omega$ = frequency of vibration (rad/sec), $i = \sqrt{-1}$, and $t_0$ = length of the digitized accelerogram (in seconds). The squared Fourier amplitude spectrum $A^2(\omega)$, indicates how the "total energy" in the earthquake motion is distributed over frequency, and its integral over all frequencies is directly related to the total motion "energy" $I_0$, or the Arias Intensity (1), as follows:

$$I_0 = \int_0^{t_0} a^2(t)\, dt = \int_{-\infty}^{\infty} a^2(t)\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} A^2(\omega) = \frac{1}{\pi}\int_0^{\infty} A^2(\omega)\, d\omega \qquad (2)$$

The equality in the center of Eq. 2 is Parseval's relation, while the equality on the right side results from the fact that $A(\omega)$ is an even function of frequency since $a(t)$ is real.

Due to the limited duration and the randomness of the phasing of contributing sinusoids, Fourier amplitude spectra of recorded ground motions appear highly variable, that is, the size and location of peaks and valleys are quite sensitive to computational details such as the choice of time and frequency intervals. The expected value of $A^2(\omega)$, obtained by appropriate local averaging or smoothing over frequency, is directly related to the spectral density function $G(\omega)$ and the duration of strong motion $s$, as follows (2):

$$G(\omega) = \frac{1}{\pi s}\, \overline{A^2(\omega)} \qquad (3)$$

$G(\omega)$ is the one-sided ($\omega \geq 0$) spectral density function which indicates how the "power" (energy per unit time) in the ground motion is distributed over frequency. The strong-motion duration $s$ is the time interval over which the "total energy" in the motion is distributed uniformly.

A well-known property of $G(\omega)$ is that its integral over all frequencies equals the variance $\sigma^2$ of the ground acceleration during the interval of strong motion:

$$\sigma^2 = \int_0^\infty G(\omega)\, d\omega \qquad (4)$$

The strong-motion duration s and the r.m.s. acceleration $\sigma$ of recorded accelerograms can be obtained in terms of the "Arias Intensity" $I_0$ ( = the time integral of the squared accelerations over the entire record length) and the absolute maximum acceleration, $a_{max}$ from the following system of two equations and two unknowns (s and $\sigma^2$):

$$I_0 = s\, \sigma^2 \qquad (5)$$

$$a_{max} = r\, \sigma \qquad (6)$$

where r is a dimensionless peak factor. The first equation states that the motion "intensity" $I_0$ is distributed uniformly, at constant average power $\sigma^2$, over the duration s. The second equation asserts that the r.m.s. acceleration and the peak acceleration are linked (probabilistically) by the peak factor r, which in turn depends very weakly, but in a predictable way, on s and on the frequency $\Omega_2$ (see Eq. 11). A simplified definition of s can be obtained by replacing r by an average value ($r \simeq 2.65$) obtained from empirical study of a large set of strong-motion accelerograms (2). Replacing $\sigma$ by $a_{max}/2.65$ in Eq. 5 yields the following (simplified) definition of strong-motion duration:

$$s \simeq (2.65)^2 (I_0/a_{max}^2) \simeq 7(I_0/a_{max}^2) \qquad (7)$$

Once the duration s has been estimated from a record, its r.m.s. strong-motion acceleration may be found from the relationship $\sigma = \sqrt{I_0/s}$.

In reality, the intensity of the ground motion first increases and then decays relatively slowly (rather than abruptly), and this can be modeled by a deterministic time-dependent variance function $\sigma^2(t)$ (which has a "box-car" shape in the idealized model presented here). To achieve the dual goal of (a) preserving the total intensity $I_0$ and (b) maintaining the relation between $a_{max}$ and the maximum of $\sigma(t)$, it is desirable that the maximum of $\sigma^2(t)$ be equal to $\sigma^2$, and that the integral of $\sigma^2(t)$ over the (longer) duration remain equal to $I_0 = \sigma^2 s$. Similarly, if one chooses to model the evolution of the frequency content of the ground motion with time, or to make duration dependent on frequency, it is desirable to maintain the time-integrated motion intensity within each frequency band, $sG(\omega)\, \Delta\omega$. These more involved stochastic descriptions which account for the "evolution" of the frequency content with time or for the frequency dependence of the duration of (different components of) the ground motion, while within the state-of-the art of stochastic modeling, present major obstacles when it comes to practical stochastic structural response analysis, adequate empirical backup for the models, and extension to account for spatial correlation decay.

Limited statistical studies were carried out based on a data set of 140 horizontal components of 70 western United States strong-motion records corresponding to different even/site pairs (2). Eleven sites (22 records) were classified as "rock" sites, and 59 sites (118 records) as "soil" sites. The mean duration for all records is 9.3 sec, and the standard deviation is 8.7 sec. For the 32 records with a "near-field" designation, the mean and the standard deviation are 6.3 and 5.5 sec, respectively. For the "far-field" records, the mean is 10.2 sec, and the standard deviation is 9.4 sec. For the records on "rock", the mean duration is 5.1 sec, and for the records on "soil", it is 10.1 sec. Some further information, including various regressions on distance and magnitude, may be found in References 2 and 3.

## THE SPECTRAL DENSITY FUNCTION: PROPERTIES AND MODELS

### The "Point" Spectral Density Function: Parameters Related to Moments

The spectral density function $G(\omega)$ expresses how the ground acceleration intensity at a given point in space is distributed over frequency. We have seen that, for actual records, $G(\omega)$ is directly connected to the "Arias intensity" $I_0$ and, through the relationship $I_0 = \sigma^2 s$, to the r.m.s. ground acceleration $\sigma$. Of course, from basic theory of stationary random processes, the principal property of $G(\omega)$ is that its integral over frequency equals $\sigma^2$. (Throughout this paper attention is restricted to a single component of ground motion.) Scaling $G(\omega)$ with respect to $\sigma^2$ yields the unit-area spectral density function,

$$g(\omega) = \frac{1}{\sigma^2} G(\omega) \tag{8}$$

and the normalized cumulative spectral distribution function:

$$f(\omega) = \int_0^{\infty} g(u)\, du \tag{9}$$

It is useful to note the analogy between the unit-area s.d.f. and the probability density function (p.d.f.) of any random variable: both are nonnegative and have unit area. The moments of the spectral density function $G(\omega)$ are:

$$\lambda_i = \int_0^{\infty} \omega^i\, G(\omega) d\omega \tag{10}$$

in which $\lambda_i$ is the ith moment, and $\lambda_0 = \sigma^2$. An array of spectral parameters $\Omega_i$, $i = 1, 2, \ldots$ is defined as follows:

$$\Omega_i = (\lambda_i/\lambda_0)^{1/i} \qquad i = 1, 2, \ldots \tag{11}$$

$\Omega_1$ is analogous to the mean and $\Omega_2$ to the root-mean-square (r.m.s.) of a random variable. A convenient measure of the spread of the dispersion of the s.d.f. about its center frequency is the bandwidth factor

$$\delta = \sqrt{1 - \lambda_1^2/\lambda_0\lambda_2} = \sqrt{\Omega_2^2 - \Omega_1^2} \; / \; \Omega_2 \tag{12}$$

which is dimensionless, always lies between 0 and 1, and increases with increasing spectral bandwidth. These spectral parameters embody very useful information about patterns of fluctuation in the time domain; for interpretation and practical use in earthquake engineering, see Ref. 4.

Effect of Local Spatial Averaging

Earthquake ground motion varies in space as well as with time. From an engineering standpoint, the distances involved may range from tens of centimeters to several kilometers to cover the dimensions of the base of strong-motion instruments as well as foundations for buildings, bridges, dams, or components of lifeline systems. Initial results from ongoing research indicate that analytical models of homogeneous random field theory (5) can be used to represent the space-time character of ground motion in a (locally) homogeneous random medium (say, a particular type of bedrock or a layer of alluvial soil) during the strong phase of an earthquake. In a wide alluvial basin, waves will tend to propagate in all directions, and the resulting random field of ground motions (say, a specified component of horizontal motion) may exhibit an isotropic spatial correlation function. (A more general "ellipsoidal" random field is characterized by a correlation function with ellipsoidal iso-correlation contours). In addition, it may be appropriate (especially in the near field) to introduce a deterministic phase lag to account for partially predictable wave front propagation.

Some relevant new results (5) of random field theory are now summarized, with emphasis on the effect of spatial averaging on the "point" s.d.f. $G(\omega)$. A two-dimensional homogeneous space-time process $x(u,t)$ may be characterized by temporal and spatial scales of fluctuation, $\theta^t$ and $\theta^u$, respectively, and by a scalar space-time correlation measure $\alpha = c_\alpha \theta^t \theta^u$, where $c_\alpha$ is a dimensionless constant greater than or equal to one. Decomposition of the process in the frequency domain leads to the introduction of the frequency-dependent spatial scale of fluctuation $\theta_\omega^u$ which measures the spatial correlation decay of individual sinusoidal contributions to the (composite) ground motion. It is found that $\theta_\omega^u$ decreases monotonically with frequency; its largest value occurs at the frequency origin (where it equals $\alpha/\theta^t$) and it varies approximately in proportion to $\omega^{-1}$ at frequencies exceeding the "corner frequency" $\Omega_1^u$. The temporal scale of fluctuation $\theta^t$ is related to the unit-area spectral density function, $g(\omega) = G(\omega)/\sigma^2$, as follows:

$$\theta^t = \pi g(0) = \pi \frac{G(0)}{\sigma^2} \tag{13}$$

(The definition in terms of the spectrum at $\omega = 0$ applies to theoretical models of the s.d.f. and characterizes the underlying stationary stochastic process; owing to the limited duration "s" of actual accelerograms, it is appropriate to estimate $\theta^t$ based on the unit-area s.d.f. evaluated in the neighborhood of, say, $\omega = 4\pi/s$). For earthquake ground acceleration, the unit-area spectral density function $g(\omega)$ tends to be dispersed over a relatively wide band of frequencies. For such processes, the temporal scale of fluctuation $\theta^t$ contains information about the behavior of $g(\omega)$ at both high and low frequencies: at low frequencies $g(\omega) \approx \theta^t/\pi$, while $g(\omega)$ must all but vanish at frequencies exceeding $2\pi/\theta^t$.

The spatial scale of fluctuation $\theta^u$ measures ground motion correlation distances in space. Assuming the wave patterns travel (randomly in all directions) at an apparent propagation velocity V, one might expect $\theta^u = V\,\theta^t$. As we shall see, in the next section, for the Kanai-Tajimi s.d.f. with parameters $\omega_g = 4\pi$ and $\zeta_g = 0.3$, one obtains $\theta^t = 0.07$ seconds; taking V = 4000 m/s then yields $\theta^u = 280$ m. Empirical analysis of data from the SMART-I strong-motion accelerograph decay is presently underway to obtain empirical estimates of $\theta^u$.

The frequency-dependent spatial scale of fluctuation $\theta^u_\omega$ and the "composite" spatial scale $\theta^u$ are related as follows (5):

$$\theta^u = \int_0^\infty \theta^u_\omega \, g(\omega)\, d\omega \qquad (14)$$

A crude estimate for the "corner frequency" $\Omega_1^u$ (below which $\theta^u_\omega \approx c_\alpha \theta^u$ and above which $\theta^u_\omega \propto \omega^{-1}$) of wide-band processes is $\Omega_1^u \approx 0.3\ \pi/\theta^t$.

It is shown in Ref. 5 that the basic information $\theta^u$ and $\Omega_1^u$ suffices to calculate "admittance functions" $\gamma_D(\omega)$ which should be multiplied by the "point" spectral density function $G(\omega)$ in order to obtain the spectral density function $G_D(\omega)$ of the local spatial average of the component of the ground motion over an interval D (characterizing for example, the dimension of a rigid foundation slab):

$$G_D(\omega) = G(\omega)\ \gamma_D(\omega) \qquad (15)$$

The function $\gamma_D(\omega)$ depends on the ratio $D/\theta^u$ and the corner frequency $\Omega_1^u$. Its principal effect is to suppress the high frequency content of the "point" s.d.f. $G(\omega)$. It is also possible to generate cross-spectral density functions of seismic inputs at two different support points, or of two local spatial averages of the random field of ground motions over different regions in space. Although the above results apply to the case of a single spatial coordinate, similar results are available for homogeneous random fields and averaging regions in two and three dimensions (5).

Some Useful Models for $G(\omega)$

Model 1: The simplest physically realizable form of $G(\omega)$ corresponds to a band-limited white noise, for which the spectral density is constant from 0 to $\omega_1$, as follows:

$$G(\omega) \begin{cases} = G & 0 \le \omega \le \omega_1 \\ = 0 & \omega > \omega_1 \end{cases} \qquad (16)$$

The variance $\sigma^2$ equals the integral over frequency of $G(\omega)$. For band-limited noise, we have

$$\sigma^2 = G\,\omega_1 \qquad (17)$$

The temporal scale of fluctuation is

$$\theta^t = \pi\,\frac{G(0)}{\sigma^2} = \pi/\omega_1 \qquad (18)$$

Model 2: The following model seeks to account for the fact that the decay of wave amplitude with distance depends on frequency ( anelastic attenuation):

$$G(\omega) = G_r\, e^{-\omega/\omega_r} \qquad \omega > 0 \qquad (19)$$

where $\omega_r$ = characteristic frequency which varies in inverse proportion to the epicentral distance. The parameter $G_r$ depends on the earthquake magnitude and incorporates geometric decay of amplitudes with distance. The ground motion variance is

$$\sigma^2 = \int_0^\infty G(\omega)\, d\omega = G_r\,\omega_r \qquad (20)$$

and the temporal scale of fluctuation is

$$\theta^t = \pi/\omega_r \qquad (21)$$

These expressions are identical to those of Model 1.

Model 3: Based on Kanai's study (6) of the frequency content of a limited number of recorded strong ground motions, Tajimi (7) suggested the following widely used form for the spectral density function of ground motion:

$$G(\omega) = \frac{[1 + 4\zeta_g^2\,(\omega/\omega_g^2)]G_0}{[1 - (\omega/\omega_g)^2]^2 + 4\zeta_g^2\,(\omega/\omega_g)^2} \qquad (22)$$

Sample functions of this process can be obtained by filtering "ideal white noise" ($\omega_0 = \infty$ in Eq. 16)through a simple oscillator with natural frequency $\omega_g$ and viscous damping $\zeta_g$. These parameters may be interpreted as the "predominant ground frequency" and the "effective ground damping", respectively. Of course, the effective "local soil filter" $|H(\omega)|^2$ may be multiplied by any of the models for the bedrock motion spectral density function. Also, other soil filters may be more appropriate, in particular, those that connect the motion at bedrock outcropping to the motion at the surface of (or at a point within) a soil stratum, accounting consistently for both radiation and material damping (and whose parameters are strain-compatible).

The variance of the Kanai-Tajimi spectrum is

$$\sigma^2 = \int_0^\infty G(\omega)\, d\omega = \frac{\pi G_o \omega_g}{4\zeta_g}(1 + 4\zeta_g^2) \tag{23}$$

and the temporal scale of fluctuation is

$$\theta^t = \frac{4\zeta_g}{\omega_g(1 + 4\zeta_g^2)} \tag{24}$$

For example, with a typical set of parameter values, $\omega_g = 4\pi$ and $\zeta_g = 0.3$, one obtains $\theta^t \simeq 0.07$ sec.

Sources of Information

Recorded Accelerograms: Spectral density functions and their parameters (in particular, the variance $\sigma^2$ and the scale $\theta^t$) can be estimated from actual records. Recall that the parameter $\sigma$ is closely related to the recorded peak ground acceleration, $a_{max} \cong 2.65\sigma$. Consequently, the focus is the unit-area s.d.f. $g(\omega)$. Only limited statistical analysis has been done to date to establish a comprehensive information base containing attenuation laws and measures of uncertainty . Data processing of strong-motion array recordings is just beginning to provide information about variability and spatial correlation of spectral content at closely spaced recording stations. Bolt et al. (8) propose a correlation function which decays in stages: it is a weighted sum of two exponential functions, with correlation decay distances of about 300m and 10,000m, respectively.

Geophysical Models: The bedrock acceleration spectral density $G_r$ (or $G_o$) can be related to the seismic moment Mo and the corner frequency $\omega_0$ of the Brune "source spectrum" (9, 10). The effect of epicentral distance on the bedrock s.d.f. is in part independent of frequency (geometric attenuation) and in part dependent on frequency (anelastic attenuation); these effects may be incorporated through the parameters $G_r$ (or $G_o$) and $\omega_r$, respectively. Specific numerical values in the attenuation laws are different for different seismic regions, e.g., western versus northeastern North America. In work submitted to the U. S. Nuclear Regulatory Commission, the writer (11) developed a model for $G(\omega)$ based on information about displacement "source spectra" and distance-dependence from northeastern North American events (12-14); response spectra are derived by means of random vibration analysis, first for motion on bedrock and then for motion "filtered" through a local soil overburden; finally, the major sources of uncertainty and the variability of the predicted response spectra are quantified.

Local Geology: Bedrock motion tends to have relatively uniform spectral content over a relatively wide band of frequencies. Within this frequency range, the shape of the ground motion s.d.f. at the top of a soil deposit will depend on the layer configuration and dynamic soil properties. "Basin effects" can also be incorporated in this type of representation.

Inverting Known Response Spectra: Methodology based on random vibration theory is available to predict response spectra corresponding to a given probability of being exceeded (4). Inverting this relationship yields an estimate of $G(\omega)$ if values are assumed for the duration s and the exceedance probability. Pfaffinger (15) recently derived the spectral density functions corresponding to Housner's average response spectra (16) and the standard Newmark-Blume-Kapur response spectra (17).

## USE OF THE MODEL IN SEISMIC ANALYSIS AND DESIGN

### Prediction of Multi-Degree System Response

The simplest seismic analysis procedures are based directly on response spectra. To predict linear elastic multi-degree system response by the response spectrum approach, individual modal maxima are combined, usually by calculating the square root of the sum of the squares (SRSS), to provide an estimate of the multi-degree system peak response. The method provides no information about the degree to which actual responses might deviate from the predicted value. Similar approximate procedures have been proposed to predict the response of light equipment in buildings and certain nonlinear systems directly from a set of specified smooth response spectra. The proposed stochastic representation (i.e., sudden exposure, for "s" seconds, to stationary Gaussian excitation with given s.d.f.) permits response predictions with equal ease and superior reliability compared to procedures based directly on the response spectrum (4). Whenever system behavior is highly sensitive to ground motion duration, the proposed motion representation has the obvious advantage of accounting explicitly for duration.

Random vibration methodology provides approximate closed-form predictions (fractiles of the distribution of) seismic response of multi-degree linear systems (4). In fact, the form of the expression for the multi-degree system response variance motivates improved rules for modal combination that are of immediate benefit in the conventional response spectrum approach. (It suffices to replace the modal standard deviations by the response spectra ordinates). The writer first suggested such a "stochastic modal superposition" (SMS) procedure that fully accounts for cross-correlation between modal responses (4). An alternate procedure (referred to as "complete quadratic combination",or CQC) was suggested by Der-Kiureghian (18); its derivation assumes white noise excitation, ignores the transient nature of the seismic response, and neglects secondary effects attributable to differences in the peak factors of multi- and single-degree responses. If the response is stationary and the input is white noise, the two procedures are identical, as they are simply based on two different ways of expanding the multi-degree response variance.

It should be noted that the methodology to predict response spectra and multi-degree system response when the excitation s.d.f. is prescribed also enables evaluation of the effect of local spatial averaging on system response; it suffices to replace the input "point" s.d.f. $G(\omega)$ by the s.d.f. of the local spatial average, e.g., $G_D(\omega)$ given by Eq. 15. Similarly, random field models of the earthquake ground motion provide the input for random vibration analysis of structures with multiple supports and spatially extended foundations.

A common analysis procedure is step-by-step time integration based on one or more recorded or synthetic accelerograms. The "$G(\omega)$-s" stochastic representation is a suitable starting point for ground motion simulation. As mentioned before, simulated accelerograms can be made more realistic in the variation of motion intensity-versus-time or the time-dependence of the frequency content, provided the empirical connections to $A(\omega)$, $I_0$ and $a_{max}$ (inherent in the basic representation) are maintained.

Relation to Seismic Risk and Design Criteria

An important issue is how ground motions should be characterized for seismic design purposes. If the "design event" magnitude and source-to-site distance are specified, conventional attenuation laws yield estimates of the peak ground motion amplitudes. From the peak ground acceleration, $\sigma$ can be estimated (roughly $\sigma \cong a_{max}/2.65$). A crude estimate of the quantity $\theta^t$ may be obtained from the relationship $\theta^t \cong 2\ (v_{max}/a_{max})$ for ground motion on bedrock ($a_{max}$ is in cm/sec$^2$ and $v_{max}$ is the peak ground velocity in cm/sec.) Similarly, the strong-motion duration may be estimated as a function of magnitude and distance. Of course, local soil conditions may change these values to some extent.

If an occurrence probability is associated with each magnitude-distance pair, and hence with the corresponding duration and spectral parameters (more generally, the "random field" characteristics) and the associated seismic response predictions, it becomes possible to obtain estimates of annual exceedance probability for some particular response quantity of interest. To limit the computations, one could arrange the magnitude-distance pairs into a limited number of clusters which together contribute the major part of the seismic risk. A reasonable compromise may be to specify two or three "design" conditions — preferably with associated likelihoods — each defined in terms of a specified magniture and distance.

A less cumbersome(but more compromising)alternative is to specify a "design" spectral density function that may contain artificially high variance contributions in a very wide band of frequencies. Such a s.d.f. may be obtained by inverting "design" response spectra such as the mean-plus-one-standard deviation Newmark-Blume-Kapur response spectra.

## CONCLUSIONS

It has been argued in this paper that the "G(ω)-s" stochastic model of earthquake ground motion is superior to the conventional representation based on the response spectrum. The principal features of the proposed model are:

(i) G(ω) provides direct information about the frequency content of (the strong shaking phase of) ground motions;

(ii) the model explicitly accounts for motion duration;

(iii) it has a firm empirical basis as G(ω), $\sigma^2$ and s are clearly connected with A(ω), $a_{max}$ and $I_0$;

(iv) geophysical models of ground motion provide information about the dependence of G(ω) on basic earthquake source and wave propagation parameters:

(v) it facilitates incorporating the effect of local geology on earthquake motions;

(vi) it leads to improved predictions of structural response for all types of systems and yields predictions of level crossing frequencies and cumulative damage measures associated with fatigue or liquefaction;

(vii) it permits extending ground motion models to account for (local) spatial variation;

(viii) it is compatible with conventional representations of earthquake ground motion such as the response spectrum and provides a convenient starting point for generating synthetic accelerograms.

## ACKNOWLEDGEMENT

This research is supported by the National Science Foundation under Grant No. CEE-8211021 on "Improved Ground Motion Modeling for Seismic Design". Any opinions, findings and conclusions or recommendations are those of the author and do not necessarily reflect the views of the National Science Foundation.

## REFERENCES


1. Arias, A., "A Measure of Earthquake Intensity" in Seismic Design of Nuclear Power Plants, R. Hansen, Editor, M.I.T. Press, 1970.
2. Vanmarcke, E.H. and Lai, S., "Strong-Motion Duration and R.M.S. Amplitude of Earthquake Records", Bull. Seism. Society of America, August, 1980.
3. Vanmarcke, E.H., "Representation of Earthquake Ground Motion: Scaled Accelerograms and Equivalent Response Spectra", U. S. Army Engineer Waterways Experiment Station, State-of-the-Art for Assessing Earthquake Hazards in the United States, Report 14, August, 1979.
4. Vanmarcke, E.H. (1976), "Structural Response to Earthquakes", Ch. 8 in Seismic Risk and Engineering Decisions, C. Lomnitz and E. Rosenblueth, Editors, Elsevier Publ., Amsterdam - Oxford - New York.
5. Vanmarcke, E.H., Random Fields: Analysis and Synthesis, The M.I.T. Press, Cambridge, Mass. and London, England, 1983.
6. Kanai, K., "An Empirical Formula for the Spectrum of Strong Earthquake Motions", Bull. Earthq. Res. Institute, University of Tokyo, 39, pp. 85-95, 1961.
7. Tajimi, H., "A Statistical Method of Determining the Maximum Response of a Building Structure During an Earthquake", Proc. 2nd World Conf. on Earthq. Engrg., Vol II, Tokyo and Kyoto, Japan, 1960.
8. Bolt, B.A., Tsai, Y.B., Yeh, K. and Hsu, M.K., "Earthquake Strong Motions Recorded by a Large Near-Source Array of Digital Seismographs", Earthq. Eng. and Struct. Dynamics, Vol. 10, pp. 561-73, 1982.
9. Brune, J.N., "Tectonic Stress and the Spectra of Seismic Shear Waves", J. Geophys. Res., Vol. 75, pp. 4997-5009, 1970.
10. Berrill, J.B., "A Study of High Frequency Strong Ground Motion from the San Fernando Earthquake", Ph.D. Dissertation, California Institute of Technology, March, 1975.
11. Vanmarcke, E.H., "Prediction of Site Dependent Earthquake Response Spectra", Report to Weston Geophysical Corporation and Yankee Atomic Electric Co.; submitted to the Nuclear Regulatory Commission, January, 1980.
12. Street, R.L., Herrman, R.B. and Nuttli, D.W., "Spectral Characteristics of the Lg Wave Generated by Central United States Earthquakes", Geophys. J. Roy. Astron. Soc., Vol. 41, pp. 51-63, 1975.
13. Street, R.L. and Turcotte, F.T., "A Study of Northeastern North American Spectral Moments, Magnitudes and Intensities", Bull. Seism. Soc. Am., Vol. 67, pp. 599-614, 1977.

14. Nuttli, O.W., "Seismic Wave Attenuation and Magnitude Relations for Eastern North America", J. Geophys. Res., Vol. 78, pp. 786-885, 1973.

15. Pfaffinger, D.D., "Calculation of Power Spectra from Response Spectra", J. of Engineering Mechanics, Vol. 109, No. 1, February, 1983.

16. Housner, G.W., "Behavior of Structures during Earthquakes", J. of the Engineering Mechanics Division, ASCE, Vol. 85, No. EM4, October, 1959.

17. Newmark, N.M., Blume, J.A. and Kapur, K.K., "Seismic Design Spectra for Nuclear Power Plants", J. of the Power Division, ASCE, Vol. 99, No. PO2, November, 1973.

18. Der Kiureghian, A., "Structural Response to Stationary Excitation," Journal of the Engineering Mechanics Division, ASCE, Vol. 106, No. EM6, pp. 1195-1213, December, 1980.